SSLC CLASS NOTES: PERMUTATION AND COMBINATION

# chapter - 4

# Permutation

# and

# Combination

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SSLC Class Notes: Permutation and Combination

# Fundamental principle of counting

If one activity, can be done in 'm' number of different ways, for each of these 'm' different ways, a second activity can be done in 'n' number of different ways and for each of these activities, a third activity can be done in 'p' ways, then all the three activities one after the other can be done in $(m \times n \times p)$ number of ways.

## ILLUSTRATIVE EXAMPLES

Examples1: How many 2 - digit numbers can be formed from the digits {1, 2, 3, 4, 5} without repetition and with repetition?

**Sol::** 2 Digits number formed without reetition

| Ten | Unit |
|---|---|
| 5 | 4 |

Total numbers = 5x4 = 20

With repetition:

Total numbers = 5x5 = 25

| Ten | Unit |
|---|---|
| 5 | 5 |

Example2 How many 3 letter code can be formed by using the five vowels without repetitions?

Sol:

The vowels are: a, e, i, o, u

| | | |
|---|---|---|
| 5 | 4 | 3 |

Total Codes are to be formed = 5x4x3 = 60

Example3: How many 3 - digit numbers can be formed from the digits 0, 1, 2, 3 and 4 with repetitions

| Hundred | Ten | Unit |
|---|---|---|
| 4 | 5 | 5 |

Total numbers = 4x5x5 = 100

## Exercise 4.1

1. How many 3 - digit numbers can be formed using the digits 1, 2, 3, 4, 5, 6 without repeating any digit?

| Hundred | Ten | Unit |
|---|---|---|
| 4 | 5 | 6 |

Total numbers = 4x5x6 = 120

2. How many 3 digit even numbers can be formed using the digits 3, 5, 7, 8, 9, if the digits are not repeated?

| Hundred | Ten | Unit |
|---|---|---|
| 3 | 4 | 1 |

Total numbers = 3x4x1 = 12

3. How many 3 letter code can be formed using the first 10 letters of English alphabet, if no letter can be repeated?

| | | |
|---|---|---|
| 10 | 9 | 8 |

Total code = 10x9x8 = 720

## SSLC Class Notes: Permutation and Combination

**4.** How many 5 digit telephone numbers can be formed using the digits 0 to 9, if each number starts with 65 and no digit appear more than once?

| 1st digit | 2nd digit | 3rd digit | 4th digit | 5th digit |
|---|---|---|---|---|
| 1 | 1 | 8 | 7 | 6 |

Total numbers = 1x1x8x7x6 = 336

**5.** If a coin is tossed 3 times, find the number of outcomes.

| 1st toss | 2nd toss | 3rd toss |
|---|---|---|
| **2** | 2 | 2 |

Total outcomes = 2x2x2 = 8

**6.** Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags one below the other?

| 1st Flag | 2nd Flag |
|---|---|
| **5** | 4 |

Total signals = 5x4 = 20

**Permutation**: A permutation is an ordered arrangement of a set of objects. It is an act of arrangements of objects in an orderly way.

**Combination :** A combination is a selection of a set of objects without any order. It is an act of selection of objects not involving any orderly way

## Exercise 4.2

I. Below are given situations for arrangements and selections. Classify them as examples of permutations and combinations.

1. A committee of 5 members to be chosen from a group of 12 people - **Combintation**
2. Five different subject books to be arranged on a shelf. **- Permutation**
3. There are 8 chairs and 8 people to occupy them- **Permutation**
4. In a committee of 7 persons, a chairperson, a secretary and a treasurer are to be chosen - **Permutation**
5. The owner of children's clothing shop has 10 designs of frocks and 3 of them have to be displayed in the front window - **Permutation**
6. Three-letter words to be formed from the letters in the word 'ARITHMETIC'- **Permutation**
7. In a question paper having 12 questions, students must answer the first 2 questions but may select any 8 of the remaining ones. - **Combintation**
8. A jar contains 5 black and 7 white balls. 3 balls to be drawn in such a way that 2 are black and 1 is white- **Combintation**
9. Three-digit numbers are to be formed from the digits 1, 3, 5, 7, 9 where repetitions are not allowed- **Permutation**
10. Five keys are to be arranged in a circular key ring – **Permutation**
11. There are 7 points in a plane and no 3 of the points are collinear. Triangles are to be drawn by joining three non-collinear points.- **Combination**
12. A collection of 10 toys are to be divided equally between two children.- **Combination**

## SSLC Class Notes: Permutation and Combination

# Permutation

**Number of objects –n Total number of places- r ⇒$^nP_r$**

| 1 | 2 | 3 | 4 | | r-2 | r-1 | r |
|---|---|---|---|---|---|---|---|
| **n-(1-1)** | n-(2-1) | n-(3-1) | n-(4-1) | | n-(r-2-1) | n-(r-1-1) | n- (r-1) |
| n | n-1 | n-2 | n-3 | | n-r+3 | n-r+2 | n-r+1 |
| r = n ಆಗಿದ್ದಾಗ | | | | | | | |
| n | n-1 | n-2 | n-3 | | n-n+3 | n-n+2 | n-n+1 |
| n | n-1 | n-2 | n-3 | | 3 | 2 | 1 |

## Factorial notation

n! = n(n-1)(n-2)(n-3)- - - - - -3x2x1

Example: 5!= 5x4x3x2x1

4!= 4x3x2x1

3!= 3x2x1

2!= 2x1

1!= 1

**0!= 1**

## Exercise 4.3

1. Convert the following products into factorials.

(i).1x2x3x4x5x6x7 = 7!

(ii).18x17x…………3x2x1 = 18!

(iii).6x7x8x9 = $\frac{9!}{5!}$

(iv).2x4x6x8 = (2x1)(2x2)(2x3)(2x4) = 16x4!

2. Evaluate.

(i).6! = 6x5x4x3x2x1 = 720

(ii).9! = 9x8x7x6x5x4x3x2x1 = 3,62,880

(iii).8!-5! = 8x7x6x5x4x3x2x1 – 5x4x3x2x1=40320 – 120 =40200

(iv).$\frac{7!}{5!} = \frac{7x6x5x4x3x2x1}{5x4x3x2x1} = 7x6 = 42$

(v). $\frac{12!}{(9!)(3!)} = \frac{12x11x10x9x8x7x6x5x4x3x2x1}{9x8x7x6x5x4x3x2x1x3x2x1} = \frac{12x11x10}{3x2x1} = \frac{2x11x10}{3x2x1} = 220$

(vi) $\frac{30!}{28!} = \frac{30x29x28!}{28!} = 30x29 = 870$

**3.** Evaluate : (i). $\frac{n!}{(n-r)!}$ and (ii). $\frac{n!}{(n-r)!r!}$ when n=15 ಮತ್ತು r = 2 ಆದಾಗ

(i). $\frac{n!}{(n-r)!}$

$= \frac{15!}{(15-2)!}$

$= \frac{15!}{13!} = \frac{15x14x13!}{13!}$ = 5x14 = 210

(ii). $\frac{n!}{(n-r)!r!}$

$= \frac{15!}{(15-2)2!}$

$= \frac{15!}{13!2!}$

$= \frac{15x14x13!}{13!2!}$

$= \frac{15x14}{2x1}$ = 15x7 = 105

4. Find the LCM of 4!, 5!, 6!.
   4! = 4x3x2x1
   5! = 5x4x3x2x1
   6! = 6x5x4x3x2x1
   LCM = 6x5x4x3x2x1 = 720
5. If (n+1)! = 12(n-1)! Find the value of 'n'.
   (n+1)! = 12(n-1)!

$\Rightarrow \frac{(n+1)!}{(n-1)!} = 12$

$\Rightarrow \frac{(n+1)n(n-1)!}{(n-1)!} = 12$

$\Rightarrow (n+1)n = 12$

$\Rightarrow (3+1)3 = 12$

$\Rightarrow n = 3$

## To derive the formula for $^nP_r$ In factorial notation:

$^n\mathbf{P_r}$ = n(n-1)(n-2)(n-3). . . . . . . (n-r+3)(n-r+2)(n-r+1

$\Rightarrow {}^n\mathbf{P_r} = \frac{[n(n-1)(n-2)(n-3)\ldots\ldots(n-r+3)(n-r+2)(n-r+1)][(n-r)(n-r-1)(n-r-2). . . .3x2x1]}{(n-r)(n-r-1)(n-r-2). . . .3x2x1}$

$\Rightarrow {}^n\mathbf{P_r} = \frac{n!}{(n-r)!}$ $\quad [\mathbf{r < 0 \leq n}]$

| $^n\mathbf{P_0}$ | **1** |
|---|---|
| $^n\mathbf{P_n}$ | n! |
| $^n\mathbf{P_1}$ | n |

### ILLUSTRATIVE EXAMPLES

Example1: Evaluate (i) $^7P_3$ (ii)$^8P_5$

Sol:(i) $^7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7x6x5x4!}{4!}$ = 7x6x5 = 210

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(ii) $^8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{5!} = \frac{8x7x6x5!}{5!} = 8x7x6 = 6720$

Example2: Find 'r' if $5.\,^4P_r = 6.\,^5P_{r-1}$.

$5x\frac{4!}{(4-r)!} = 6 \text{ x } \frac{5!}{[(5-(r-1)!}$

$\frac{5x4!}{(4-r)!} = \frac{6x5!}{(5-r+1)!}$

$\frac{5!}{(4-r)!} = \frac{6!}{(6-r)!}$

$\frac{(6-r)!}{(4-r)!} = \frac{6!}{5!}$

$\frac{(6-r)(5-r)(4-r)!}{(4-r)!} = \frac{6!}{5!}$

$(6-r)(5-r) = 6$

$30 - 6r - 5r + r^2 = 6$

$r^2 - 11r + 24 = 0$

$r^2 - 8r - 3r + 24 = 0$

$r(r-8) - 3(r-8) = 0$

$(r-8)(r-3) = 0$

$r = 8$ Or $r = 3$

$r = 8 > n$ not possible

$\therefore r = 3$

Example3: Prove that n! + (n+1)! = n!(n+2).

Sol:LHS:n! + (n+1)! = n! + (n+1)n!

= n!(1+n+1) = n!(n+2)=RHS

Example 4:Find 'n' if $\frac{nP_4}{n-1P_4} = \frac{5}{3}$.

$\frac{n(n-1)(n-2)(n-3)}{(n-1)(n-2)(n-3)(n-4)} = \frac{5}{3}$

$\frac{n}{(n-4)} = \frac{5}{3}$

$3n = 5(n-4)$

$3n = 5n - 20$

$2n = 20$

$n = 10$

Example:5: If $^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$ find the value of 'n'?

$\frac{^{2n+1}P_{n-1}}{^{2n-1}P_n} = \frac{3}{5}$

$5 \text{ x } ^{2n+1}P_{n-1} = 3 \text{ x } ^{2n-1}P_n$

$5x\frac{(2n+1)!}{2n+1-n+1)!} = 3 \text{ x } \frac{(2n-1)!}{2n-1-n)!}$

$5x\frac{(2n+1)!}{(n+2)!} = 3 \text{ x } \frac{(2n-1)!}{(n-1)!}$

$5x\frac{(2n+1)2n(2n-1)!}{(n+2)(n+1)n(n-1)!} = 3 \text{ x } \frac{(2n-1)!}{(n-1)!}$

$\frac{5(2n+1)2}{(n+2)(n+1)} = 3$

$\frac{10(2n+1)}{(n+2)(n+1)} = 310n + 10 = 3(n+2)(n+1)$

$20n + 10 = 3n^2 + 9n + 6$

$3n^2 - 11n - 6 = 0$

$(3n + 1)(n - 4) = 0$

$n = \frac{-1}{3}$ or $n = 4$

n is a positive integer $\Rightarrow n = 4$

Example6 (i): if ${}^{n}P_{n} = 5040$ find the value of 'n'?

${}^{n}P_{n} = 5040$

$n! = 5040$

$7x6x5x4x3x2x1 = 5040$

$\therefore n = 7$

(ii):If ${}^{n}P_{2} = 90$ find 'n'?

$n(n-1) = 10x9$

$\therefore n = 10$

(iii) If ${}^{11}P_{r} = 990$ then 'r'?

${}^{11}P_{r} = 990$

$11x10x9 = 990$

$\therefore r = 3$

## Exercise – 4.4

1. Evaluate:

(i). ${}^{12}P_{4}$

$nP_r = \frac{n!}{(n-r)!}$

${}^{12}P_{4} = \frac{12!}{(12-4)!}$

${}^{12}P_{4} = \frac{12!}{8!}$

${}^{12}P_{4} = \frac{12x11x10x9x8!}{8!}$

${}^{12}P_{4} = 12x11x10x9$

${}^{12}P_{4} = 11{,}880$

(ii). ${}^{75}P_{2}$

${}^{n}P_{r} = \frac{n!}{(n-r)!}$

${}^{75}P_{2} = \frac{75!}{(75-2)!}$

${}^{75}P_{4} = \frac{75!}{73!}$

${}^{75}P_{2} = \frac{75x74x73!}{73!}$

${}^{75}P_{2} = 75x74$

${}^{75}P_{2} = 5{,}550$

(iii). ${}^{8}P_{8}$

${}^{n}P_{n} = n!$

${}^{8}P_{8} = 8!$

${}^{8}P_{8} = 40{,}320$

(iv). ${}^{15}P_{1}$

$nP_1 = n$

${}^{15}P_{1} = 15$

${}^{15}P_{1} = 11{,}880$

(v).$38P_0$

$^{n}P_0 = 1$

$^{30}P_0 = 1$

2. (i). If $^{n}P_4 = 20\,^{n}P_2$ find 'n'

$nP_4 = 20nP_2$

$nP_r = \frac{n!}{(n-r)!}$

$\Rightarrow n(n-1)(n-2)(n-3) = 20\,n(n-1)$

$\Rightarrow (n-2)(n-3) = 20$

$\Rightarrow (7-2)(7-3) = 20$ [$\because 5x4 = 20$]

$\Rightarrow n = 7$

(ii).If $^{5}P_r = 2\,.\,^{6}P_{r-1}$ find 'r'

$nP_r = \frac{n!}{(n-r)!}$

$5P_r = 2\,.\,6P_{r-1}$

$\Rightarrow \frac{5!}{(5-r)!} = 2\,.\,\frac{6!}{[6-(r-1)]!}$

$\Rightarrow \frac{5!}{(5-r)!} = 2\,.\,\frac{6x5!}{[6-r+1]!}$

$\Rightarrow \frac{5!}{(5-r)!} = 2\,.\,\frac{6x5!}{(7-r)!}$

$\Rightarrow \frac{5!}{(5-r)!} = 2\,.\,\frac{6x5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow 1 = 2\,.\,\frac{6}{(7-r)(6-r)}$

$\Rightarrow (7-r)(6-r) = 2\,X6$

$\Rightarrow (7-r)(6-r) = 12$

$\Rightarrow (7-3)(6-3) = 12$ [$\because 4X3=12$]

$\Rightarrow r = 3$

3. If $^{n}P_4 : {}^{n}P_5 = 1:2$ find 'n'.

$nP_4 : nP_5 = 1:2$

$\Rightarrow \frac{nP_4}{nP_5} = \frac{1}{2}$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)(n-4)} = \frac{1}{2}$

$\Rightarrow \frac{1}{(n-4)} = \frac{1}{2}$

$\Rightarrow 2 = (n-4)$

$\Rightarrow n = 6$

4. $^{9}P_5 + 5.\ ^{9}P_4 = {}^{10}P_r$

$\Rightarrow 9x8x7x6x5 + 5(9x8x7x6) = 10P_r$

$\Rightarrow 9x8x7x6x5 + 9x8x7x6x5 = 10P_r$

$\Rightarrow 2(9x8x7x6x5) = 10P_r$

$\Rightarrow 9x8x7x6x10 = 10P_r$ [$\because 5x2 = 10$ ]

$\Rightarrow 10x9x8x7x6 = 10P_5$ [$\because$ 5 terms]

$\Rightarrow r = 5$

## Application questions on Permutation(Examples)

## SSLC Class Notes: Permutation and Combination

Example1: 6 songs are to be rendered in a programme. In how many different orders could they be performed?

Sol: 6 songs are to be rendered is: $^6P_6 = 6!$

$^6P_6 = 6! = 6x5x4x3x2x1 = 720$

Example2: How many words (with or without dictionary meaning) can be made from the letters in the word LASER assuming that no letter is repeated it, such that LASER

(i) All letters are used at a time

(ii) 3 letters are used at a time

(iii) All letters are used such that it should begin with letter A and end with letter R

Sol:Formula $^nP_r = \frac{n!}{(n-r)!}$

(i) $^5P_5 = 5! = 120$

(ii) $^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5X4X3 = 60$

(iii)

| Starting with letter A | Remaining letters | Ending with letter R |
|---|---|---|
| $^1\mathbf{P_1}$ | $^3P_3$ | $^1P_1$ |

$^1P_1x\ ^3P_3x^1P_1 = 1x3!\ x1 = 3x2x1 = 6$

Example3: In how many ways can 7 different books be arranged on a shelf? In how many ways three particular books are always together?

Sol:7 The 7 books can be arranged in:$^7P_7$

$^7P_7 = 7! = 7x6x5x4x3x2x1 = 5040$

Since three particular books are always together, let us tie three books together and then consider them as one book (or one unit). Remaining four books have to be considered separately. So in all we can consider 7 books as

| 1 | 2 | 3 | 4 | 5 | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $^5P_5\ x\ ^3P_3$ | | | | | | |

These 5 books can be arranged in $^5P_5$ ways

5P5 x 3P3 = 5!x3! = 120 x 6 = 720

Example4: How many five digit numbers can be formed with the digits {2, 3, 5, 7, 9} which lie between 30,000 and 90,000 using each digit only once?

Sol:

| 10 thousand | Thousand | Hundred | Ten | Unit |
|---|---|---|---|---|
| Possible digits | | | | |
| 3,5,7 | 2,3,5,7,9 | 2,3,5,7,9 | 2,3,5,7,9 | 2,3,5,7,9 |
| 3 | 4 | 3 | 2 | 1 |

Total numbers: 3x4x3x2x1 = 72

Example5: A DNA molecule will have a nitrogen base which consists of different bases A, G, T or C all attached to it? In how many ways the three bases can be arranged without repetition?

Sol:$^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4! = 24$

## Exercise4.5

1. How many words with or without dictionary meaning can be formed using all the letters of the word 'JOULE' using each letter exactly once?

   $^nP_r = \frac{n!}{(n-r)!}$

   n = 5, r = 5

   $^5P_5 = \frac{5!}{(5-5)!}$ = 5! = 5x4x3x2x1 = 120

2. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

   Sol:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| **M** | F | M | F | M | F | M | F | M |

   Men can be seated 1,3,5,7,9th places in $^5P_5$ ways

   Women can be seated 2,4,6,7th places in- $^4P_4$ways.

   ∴ By fundamental principle of counting men and women together seated in $5P_5 x 4P_4$ways

   $^5P_5 x ^4P_4$

   $^nP_n$ = n!

   $\Rightarrow ^5P_5 x 4P_4$ =5! x4!

   =120 x24

   **=2880**

3. In how many ways can 6 women draw water from 6 wells, if no well remains unused?

   6 Women draw water from 6 wells in $^6P_6$ ways.

   $^nP_n$ = n!

   $\Rightarrow ^6P_6$ = 6!

   =720

4. 8 students are participating in a competition. In how many ways can the first three prizes be won?

   The possibility of the first three prizes be won by 3 students out of 8 - $^8P_3$

   $^nP_r = \frac{n!}{(n-r)!}$

   $\Rightarrow ^8P_3 = \frac{8!}{(8-3)!}$

   $\Rightarrow ^8P_3 = \frac{8!}{5!}$

   $\Rightarrow ^8P_3$ = 8x7x6

   =336

5. Find the total number of 2- digit numbers.

   The digits are: 1,2,3,4,5,6,7,8,9,0

   The Tenth place can be filled 9 ways [except ‘0’ out 10 digits].

   The unit place can be filled in 10 ways [ repetition allowed, include ‘0’ also].

| Ten | Unit |
|---|---|
| 9 | 10 |

   ∴ 2- number digits = 9x10 = 90

## SSLC Class Notes: Permutation and Combination

6. How many 4- digit numbers can be formed using the digits 1, 2, 3, 7, 8, 9 (repetitions not allowed)?
   (a) How many of these are less than 6000?
   (b) How many of these are even?
   (c) How many of these end with7?
   (a)[To get number < 6000,we fill the thousand palce with numbers 1,2,and3 in3-ways]

| Thousand | Hundred | Ten | Unit |
|---|---|---|---|
| $^3P_1$ | $^5P_1$ | $^4P_1$ | $^3P_3$ |
| 3 | 5 | 4 | 3 |
| 3x5x4x3 = 180 | | | |

(b)[ T get even numbers we can fill the unit place with 2,and 8 in 2-ways]

| Thousand | Hundred | Ten | Unit |
|---|---|---|---|
| $^3P_1$ | $^4P_1$ | $^5P_1$ | $^2P_1$ |
| 3 | 4 | 5 | 2 |
| 3x4x5x2 = 120 | | | |

(c)[ The unit place to be filled with the digit 7 only]

| Thousand | Hundred | Ten | Unit |
|---|---|---|---|
| $^3P_1$ | $^4P_1$ | $^5P_1$ | $^1P_1$ |
| 3 | 4 | 5 | 1 |
| 3x4x5x1 = 60 | | | |

7. There are 15 buses running between two towns. In how many ways can a man go to one town and return by a different bus?
   Type equation here.Total numbers of ways can a man go to town and return by a different Bus
   = 15x 14 = 210

| To town | Return |
|---|---|
| $^{15}P_1$ | $^{14}P_1$ |
| 15 | 14 |

# Combination

**Combinations of 'n' distinct objects taken 'r' at a time – $^nC_r = \frac{n!}{(n-r)!r!}$**

| $^nC_r$ | $\frac{nP_r}{r!}$ | $^5C_3$ | $\frac{5P_3}{3!}$ |
|---|---|---|---|
| $^nC_0$ | 1 | $^5C_0$ | 1 |
| $^nC_1$ | n | $^5C_1$ | 5 |
| $^nC_n$ | 1 | $^5C_5$ | 1 |
| $^nC_r$ | $^nC_{n-r}$ | $^5C_3$ | $^5C_2$ |

## ILLUSTRATIVE EXAMPLES

Example1:If $^6P_r = 360$ and $^6C_r = 15$, find 'r'.
Sol: $^6C_r x r! = {}^6P_r$
15r! = 360

$r! = \frac{360}{15}$

$r! = 24 = 4!$

$r = 4$

Example2:Prove that $^nC_r = {}^nC_{n-r}$.

Sol: $^nC_r = \frac{n!}{(n-r)!r!}$

In the formula substitute (n-r) instead of r'

$^nC_{n-r} = \frac{n!}{[n-(n-r)]!(n-r)!}$

$^nC_{n-r} = \frac{n!}{[n-n+r]!(n-r)!}$

$^nC_{n-r} = \frac{n!}{r!(n-r)!} = {}^nC_r$

Exmple3: If $^nC_8 = {}^nC_{12}$ find 'n'.

Sol: $^nC_8 = {}^nC_{12}$

$^nC_8 = {}^nC_{n-12}$

$8 = n - 12$

$n = 8 + 12$

$\therefore n = 20$

## Exercise 4.6

1. Evaluate. (i) $^{10}C_3$ (ii) $^{60}C_{60}$ (iii) $^{100}C_{97}$

(i) $^{10}C_3$

$^nC_r = \frac{n!}{(n-r)!r!}$

$^{10}C_3 = \frac{10!}{(10-3)!3!}$

$^{10}C_3 = \frac{10!}{7!3!}$

$^{10}C_3 = \frac{10x9x8}{3x2x1}$

$^{10}C_3 = 10x3x4$

$^{10}C_3 = 120$

(ii) $^{60}C_{60}$

$^nC_n = 1$

$^{60}C_{60} = 1$

(iii) $^{100}C_{97}$

$^nC_r = \frac{n!}{(n-r)!r!}$

$^{100}C_{97} = {}^{100}C_3 = \frac{100x99x98}{3x2x1}$

$^{100}C_{97} = 100x33x49$

$^{100}C_{97} = 1{,}61{,}700$

2. (i). If $^nC_4 = {}^nC_7$ find 'n'. (ii). If $^nP_r = 840$, $^nC_r = 35$ find 'n'.

(i). If $^nC_4 = {}^nC_7$ find 'n'.

$nC_r = nC_{n-r}$

$\Rightarrow r = n - r$

$\Rightarrow 4 = n - 7$

$\Rightarrow n = 11$

(ii). If $^nP_r = 840$, $^nC_r = 35$ find 'n'

$nC_r x r! = {}^nP_r$

$35\ x r! = 840$

$r! = \frac{840}{35}$

$r! = 24$

$r! = 4!$

$\therefore\ r = 4$

3. If ${}^{2n}C_3 : {}^{n}C_3 = 11:1$ find 'n'.

$\Rightarrow \frac{2n(2n-1)(2n-2)}{6} : \frac{n(n-1)(n-2)}{6} = 11:1$

$\Rightarrow \frac{4n(2n-1)(n-1)}{6} : \frac{n(n-1)(n-2)}{6} = 11:1$

$\Rightarrow \frac{4(2n-1)}{6} : \frac{(n-2)}{6} = 11:1$

$\Rightarrow \frac{\frac{2(2n-1)}{3}}{\frac{(n-2)}{6}} = \frac{11}{1}$

$\Rightarrow \frac{2(2n-1)}{3} X \frac{6}{(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{4(2n-1)}{(n-2)} = \frac{11}{1}$

$\Rightarrow 4(2n-1) = 11(n-2)$

$\Rightarrow 8n - 4 = 11n - 22$

$\Rightarrow 18 = 3n$

$\Rightarrow n = 6$

4. Verify that ${}^8C_4 + {}^8C_5 = {}^9C_4$

${}^nC_r = \frac{n!}{(n-r)!r!}$

LHS ${}^8C_4 = \frac{8!}{(8-4)!4!}$

${}^8C_4 = \frac{8!}{4!4!}$

${}^8C_4 = \frac{8x7x6x5}{4x3x2x1}$

${}^8C_4 = 7x2x5$

${}^8C_4 = 70$

${}^8C_5 = \frac{8!}{(8-5)!5!}$

${}^8C_5 = \frac{8!}{3!5!}$

${}^8C_5 = \frac{8x7x6}{6}$

${}^8C_5 = 56$

$\therefore$ LHS = 70 + 56 = 126

RHS $= {}^9C_4$

${}^9C_4 = \frac{9!}{(9-4)!4!}$

${}^9C_4 = \frac{9!}{5!4!}$

${}^9C_4 = \frac{9x8x7x6}{24}$

## SSLC Class Notes: Permutation and Combination

$^9C_4 = 9 x 7 x 2$
RHS $^9C_4 = 126$
∴ LHS = RHS

5. Prove that $\frac{nC_r}{n-1C_{r-1}} = \frac{n}{r}$ where $1 < r \leq n$

$^nC_r = \frac{n(n-1)!}{(n-r)!r(r-1)!}$

$\Rightarrow {}^nC_r = \frac{n}{r} \cdot \frac{(n-1)!}{(n-r)!(r-1)!}$

$^{n-1}C_{r-1} = \frac{(n-1)!}{(n-r)(r-1)!}$

$\frac{nC_r}{n-1C_{r-1}} = \frac{\frac{n}{r} \cdot \frac{(n-1)!}{(n-r)!(r-1)!}}{\frac{(n-1)!}{(n-r)(r-1)!}}$

$\frac{nC_r}{n-1C_{r-1}} = \frac{n}{r} \cdot \frac{(n-1)!}{(n-r)!(r-1)!} X \frac{(n-r)(r-1)!}{(n-1)!}$

$\Rightarrow \frac{nC_r}{n-1C_{r-1}} = \frac{n}{r}$

## Example problems (Page-83)

Example1: A man has 6 friends. In how many ways can he invite one or more of them to a party?

| Number of ways invite one or more | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| $^6C_1$ | $^6C_2$ | $^6C_3$ | $^6C_4$ | $^6C_5$ | $^6C_6$ |
| **6** | $\frac{6x5}{2}$ | $\frac{6x5x4}{6}$ | $\frac{6x5}{2}$ | 6 | 1 |

Total number of ways can he invite one or more:
6+15+20+15+6+1=63

Example2: For a set of 5 true or false questions no student has written all correct answers and no two students have given the same sequence of answers. What is the maximum number of students in the class for this to be possible?
Sol: Each question can be answered in exactly 2 ways either T or F
∴ Total number of ways in which the 5 questions can be answered: 2 x 2 x 2 x 2 x 2 = 32
Out of these 32 ways exactly one will be all correct answer. Since no student has written all correct answers
The maximum possible number of students = 32 – 1 = 31

Example: 3 4 friends shake hands mutually. Find the number of handshakes.
Number of handshakes will be $^nC_2$
Here n = 4,

$^4C_2 = \frac{4!}{(4-2)!2!}$

$^4C_2 = \frac{4!}{2!2!}$

$^4C_2 = \frac{4x3}{2} = 6$

∴ Number of handshakes = 6

Example4: Everybody in a function shakes hand with everybody else. The total number of handshakes is 45. Find the number of persons in the function.

Sol: $^nC_2 = 45$


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$\Rightarrow \frac{n!}{(n-2)!2!} = 45$
$\Rightarrow \frac{n(n-1)}{2} = 45$
$\Rightarrow n(n-1) = 45x2$
$\Rightarrow n(n-1) = 90$
$\Rightarrow n(n-1) = 10(10 - 1)$
$\Rightarrow n = 10$

Example5: How many committees of five with a given chairperson can be selected from 12 persons?

Sol: Number of ways

| Selecting chairperson | Selecting remaining 4 members |
|---|---|
| $^{12}C_1 = 12$ | $^{11}C_4 = \frac{11x10x9x8}{4x3x2x1} = 330$ |

∴ The possible number of such Committees= 12x330 = 3960

Example6: There are 8 points such that any of 3 of them are non collinear. How many straight lines can be drawn by joining these points?

Sol: Total number of lines that can be drawn out of 'n' non-collinear points = $^nC_2$
Here n = 8
$\Rightarrow {}^8C_2 = \frac{8x7}{2} = 28$ [ $^nC_r = \frac{n!}{(n-r)!r!}$ ]

Example7: There are 10 points such that any 3 of them are no collinear. How many triangles can be formed by joining these points?

**Sol:** Total number of triangles that can be drawn out of 'n' non-collinear points = $^nC_3$
Here n = 10
Formula: $^nC_r = \frac{n!}{(n-r)!r!}$
$\Rightarrow {}^{10}C_3 = \frac{10x9x8}{6} = 120$

Example8: How many diagonals can be drawn in a hexagon?

Sol: Total number of sides and diagonals:= $^nC_2 - n$
Formula: $^nC_r = \frac{n!}{(n-r)!r!}$
Here n = 6
$\Rightarrow {}^nC_2 - n = {}^6C_2 - 6$
= 15 – 6 = 9
∴The number of diagonals can be drawn in a hexagon = 9

Example9: The maximum number of diagonals in a polygon is 14. Find the number of sides?

**Sol:** Number of diagonals in a polygon: $= \frac{n(n-3)}{2}$
$14 = \frac{n(n-3)}{2}$
$\Rightarrow n(n-3) = 28$
$\Rightarrow n^2 - 3n = 28$
$\Rightarrow n(n-3) = 28$
$\Rightarrow n(n-3) = 7(7-3)$
$\Rightarrow n = 7$
∴ The number of sides = 7

## SSLC Class Notes: Permutation and Combination

### Exercise 4.7

1. Out of 7 Consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

3 consonants are to be selected from 7: - $^{7}C_3$

$^{n}C_r = \frac{n!}{(n-r)!r!}$

$^{7}C_3 = \frac{7!}{(7-3)!3!} = \frac{7x6x5}{3!} = 7x5 = 35$

2 vowels to be selected from 4:- $^{4}C_2$

$^{4}C_2 = \frac{4!}{(4-2)!2!} = \frac{4x3}{2!} = 6$

Number of words to be formed from the selected 3 cosonants 3 and 2 vowels:

$= {}^{7}C_3 \ x {}^{4}C_2 \ x \ 5!$

=35x6x120

**=25,200**

2. In how many ways can 5 sportsmen be selected from a group of 10?

$^{10}C_5 = \frac{10!}{(10-5)!5!}$

$^{10}C_5 = \frac{10!}{5!5!}$

$^{10}C_5 = \frac{10x9x8x7x6}{5x4x3x2x1}$

$^{10}C_5 = 2x9x7x2$

$^{10}C_5 = 252$

3. In how many ways a cricket team of eleven be selected from 17 players in which 5 players are bowlers and the cricket team must include 2 bowlers?

If we need 2 bowlers in 11 players team, must select 2 bowlers from 5

2 bowlers to be selected from 5 in - $^{5}C_2$ ways

$^{n}C_r = \frac{n!}{(n-r)!r!}$

$^{5}C_2 = \frac{5!}{3!2!} = \frac{5x4}{2} = 10$

Remaining 9 players to be selected from remaing 12 players in $12C_9$ ways

$^{12}C_9 = \frac{12!}{3!9!} = \frac{12x11x10}{6} = 2x11x10 = 220$

∴ Total number of ways the cricket team to be selected = $^{5}C_2 \ x \ ^{12}C_9$

= 10x220

**= 2,200**

4. How many (i) lines (ii) triangles can be drawn through 8 points on a circle ?

(i) The number of lines can be drawn through 8 points on a circle - $^{8}C_2$

$^{n}C_r = \frac{n!}{(n-r)!r!}$

$^{8}C_2 = \frac{8!}{6!2!} = \frac{8x7}{2} = 28$

(ii) Number of triangles can be drawn through 8 points on a circle- $^{8}C_3$

$^{n}C_n = \frac{n!}{(n-r)!r!}$

$^{8}C_3 = \frac{8!}{5!3!} = \frac{8x7x6}{6} = 56$

5. How many diagonals can be drawn in a (i) decagon (ii) icosagon?

Number of diagonals in a polygon: $= \frac{n(n-3)}{2}$

(i) Number of diagonals in a decagon $= \frac{10(10-3)}{2} = \frac{10x7}{2} = \frac{70}{2} = 35$

(ii) Number of diagonals in a icosagon $= \frac{20(20-3)}{2} = \frac{20x17}{2} = \frac{340}{2} = 170$

6. A Polygon has 44 diagonals. Find the number of sides.

Number of diagonals in a polygon: $= \frac{n(n-3)}{2}$

$44 = \frac{n(n-3)}{2}$

n(n-3) = 88 ⇒11(11-3) = 88 [ ∵ 11x8 = 88]

**⇒n = 11**

7. If there are 6 periods in each working day of a school, in how many ways can one arrange 6 subjects such that each subject is allowed at least one period?

6 subjects can be arranged in 6 piriods is $^6P_6$ ways

$^nP_n = n!$

$^6P_6 = 6!$

$^6P_6 = 720$

8. A committee of 5 is to be formed out of 6 men and 4 ladies. In how many ways can this be done when

(i) at least 2 ladies are included.

(ii) at most 2 ladies are included?

(i). A committee of 5 is to be formed out of 6 men and 4 ladies when at least 2 ladies are included

$^6C_3 x\ ^4C_2 + {}^6C_2 x\ ^4C_3 + {}^6C_1 x\ ^4C_4$ ways

$^nC_n = \frac{n!}{(n-r)!r!}$

$^6C_3 x\ ^4C_2 + {}^6C_2 x\ ^4C_3 + {}^6C_1 x\ ^4C_4$

$^6C_3 = \frac{6!}{(6-3)!3!} = \frac{6!}{3!3!} = \frac{6x5x4}{6} = 20$

$^4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{4x3}{2} = 6$

$^6C_2 = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6x5}{2} = 15$

$^4C_3 = \frac{4!}{(4-3)!3!} = \frac{4!}{1!3!} = \frac{4}{1} = 4$

$^6C_1 = \frac{6!}{(6-1)!1!} = \frac{6!}{5!1!} = \frac{6}{1} = 6$

$^4C_4 = 1$

$\Rightarrow {}^6C_3 x\ ^4C_2 + {}^6C_2 x\ ^4C_3 + {}^6C_1 x\ ^4C_4 = 20x6 + 15x4 + 6x1$

=120 + 60 + 6 **=186**

(ii). A committee of 5 is to be formed out of 6 men and 4 ladies when at most 2 ladies are included

$^6C_5 x\ ^4C_0 + {}^6C_4 x\ ^4C_1 + {}^6C_2 x\ ^4C_2$ ways

$^nC_n = \frac{n!}{(n-r)!r!}$

$^6C_5 x\ ^4C_0 + {}^6C_4 x\ ^4C_1 + {}^6C_2 x\ ^4C_2$

$^6C_5 = \frac{6!}{(6-5)!5!} = \frac{6!}{1!5!} = \frac{6}{1} = 6$

$^4C_0 = 1$

$^6C_4 = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{6x5}{2} = 15$

$^4C_1 = \frac{4!}{(4-1)!1!} = \frac{4!}{3!1!} = 4$

$^6C_2 = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6x5}{2} = 15$

$^4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{4x3}{2} = 6$

$\Rightarrow {}^6C_5 x\, {}^4C_0 + {}^6C_4 x\, {}^4C_1 + {}^6C_2 x\, {}^4C_2$ = 6x1 + 15x4 + 15x6

= 6 + 60 + 90 **= 156**

9. A sports team of 11 students is to be constituted choosing at least 5 from class IX and at least 5 from class X. If there are 8 students in each of these classes, in how many ways can the team be constituted?

A sports team of 11 students is to be constituted choosing at least 5 from class IX and at least 5 from class X is - $^8C_6 x\, {}^8C_5 + {}^8C_5 x\, {}^8C_6$ ways

$^nC_n = \frac{n!}{(n-r)!r!}$

$^8C_6 x\, {}^8C_5 + {}^8C_5 x\, {}^8C_6$

$^8C_6 = \frac{8!}{(8-6)!6!} = \frac{8!}{2!6!} = \frac{8x7}{2} = 28$

$^8C_5 = \frac{8!}{(8-5)!5!} = \frac{8!}{3!5!} = \frac{8x7x6}{6} = 56$

$\Rightarrow {}^8C_6 x\, {}^8C_5 + {}^8C_5 x\, {}^8C_6$ = 28x56 + 56x28

= 1568 +1568

**= 3,136**

10. From a group of 12 students, 8 are to be chosen for an excursion. There are 3 students who decide that either of them will join or none of them will join. In how many ways can the 8 be chosen?

From a group of 12 students, 8 are to be chosen for an excursion such that either of them will join from 3 students is $-3C_1$x $9C_7$ ways

From a group of 12 students, 8 are to be chosen for an excursion such that none of them will join from 3 students is $-3C_0$x $9C_8$ ways

∴ 8 students can be selected from 12 is - $^3C_1 x\, {}^9C_7 + {}^3C_0 x\, {}^9C_8$ ways

$^nC_n = \frac{n!}{(n-r)!r!}$

$^3C_1 = 3$ [$\because nC_1 = n$]

$^9C_7 = \frac{9!}{(9-7)!7!} = \frac{9!}{2!7!} = \frac{9x8}{2} = 36$

$^3C_0 = 1$ [$\because nC_0 = 1$]

$^9C_8 = 9$ [$\because {}^9C_8 = {}^9C_1 = 9$]

$\Rightarrow {}^3C_1 x\, {}^9C_7 + {}^3C_0 x\, {}^9C_8$ = 3x36 + 1x9

= 108 + 9

= 117